CFA - practice

Confirmatory Factor Analysis in R

Intro

Today's goal:

Teach how to do Confirmatory Factor Analysis in R.

Outline:

- Example

CFA

Confirmatory Factor Analysis

Example

twq.dat, variables:

- cgraph: inspectability (0: list, 1: graph)
- citem-cfriend: control (baseline: no control)
- cig (citem * cgraph) and cfg (cfriend * cgraph)
- s1-s7: satisfaction with the system
- q1-q6: perceived recommendation quality
- c1-c5: perceived control
- u1-u5: understandability

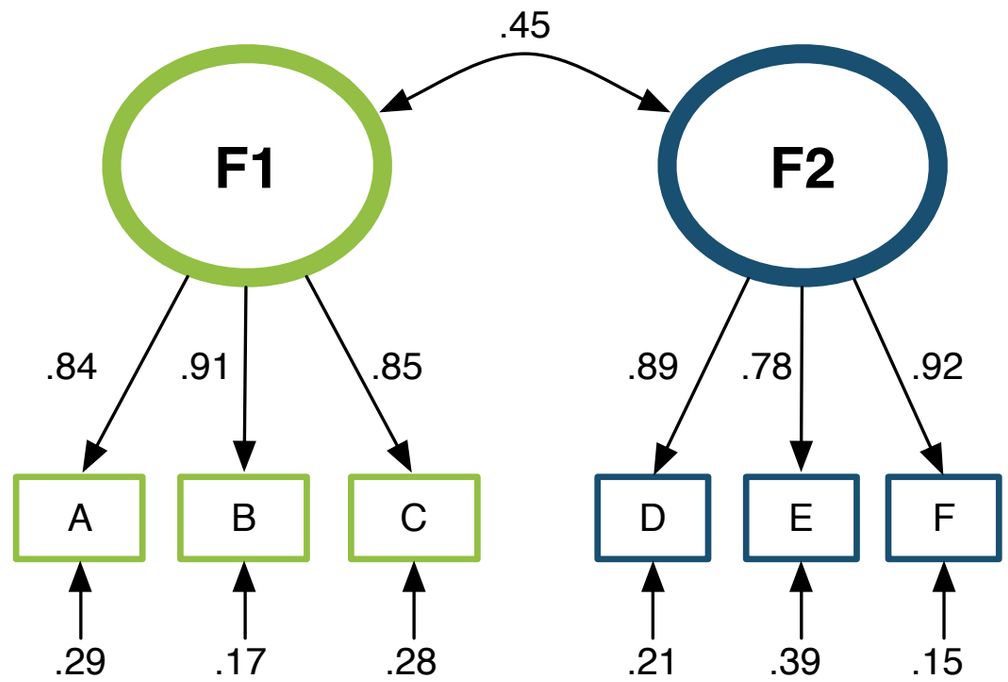
Example

twq.dat, variables:

- e1-e4: user music expertise
- t1-t6: propensity to trust
- f1-f6: familiarity with recommenders
- average rating of, and number of known items in, the top 10
- time taken to inspect the recommendations

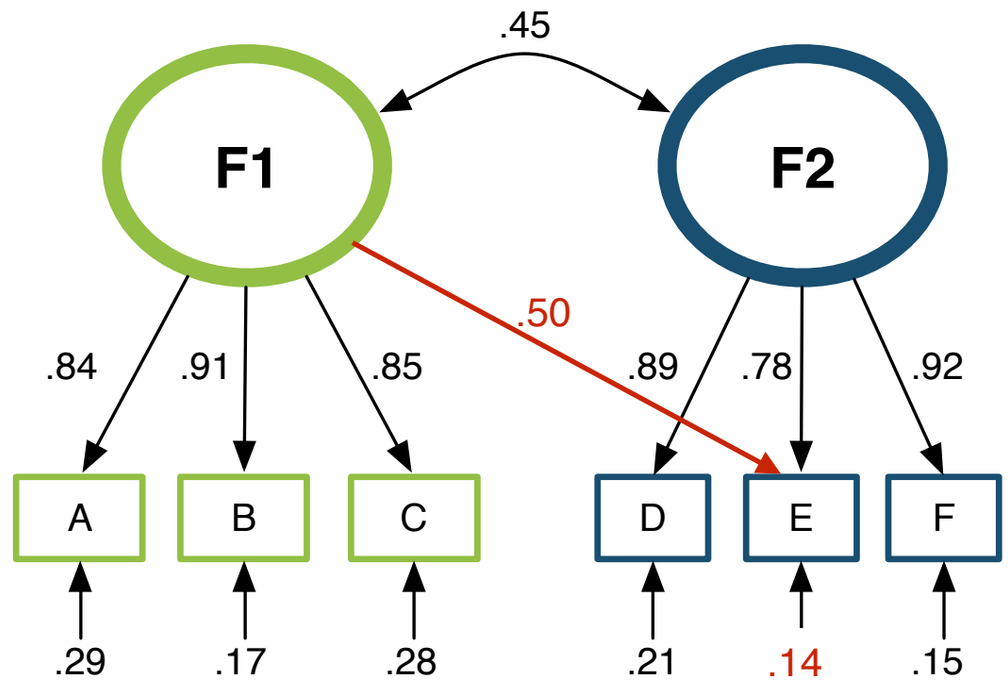
CFA syntax

```
model <- '
    F1 =~ A+B+C
    F2 =~ D+E+F
'
```



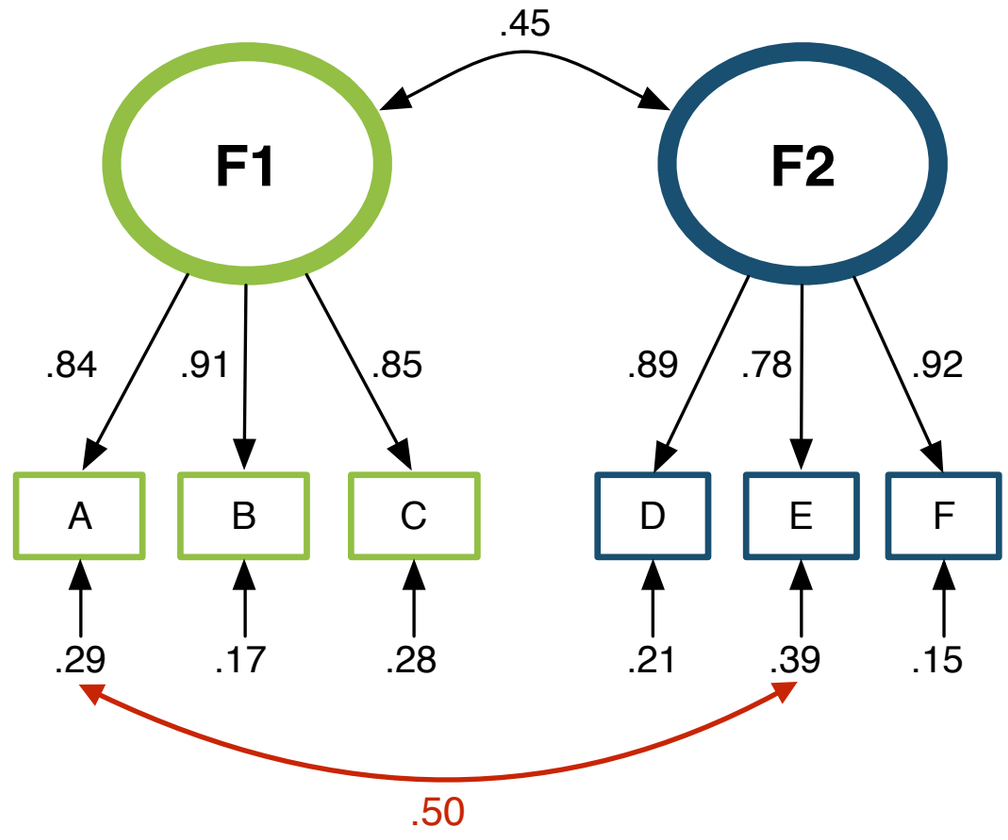
CFA syntax

```
model <- '
    F1 =~ A+B+C+E
    F2 =~ D+E+F
'
```



CFA syntax

```
model <- '
    F1 =~ A+B+C
    F2 =~ D+E+F
    A ~~ E
'
```

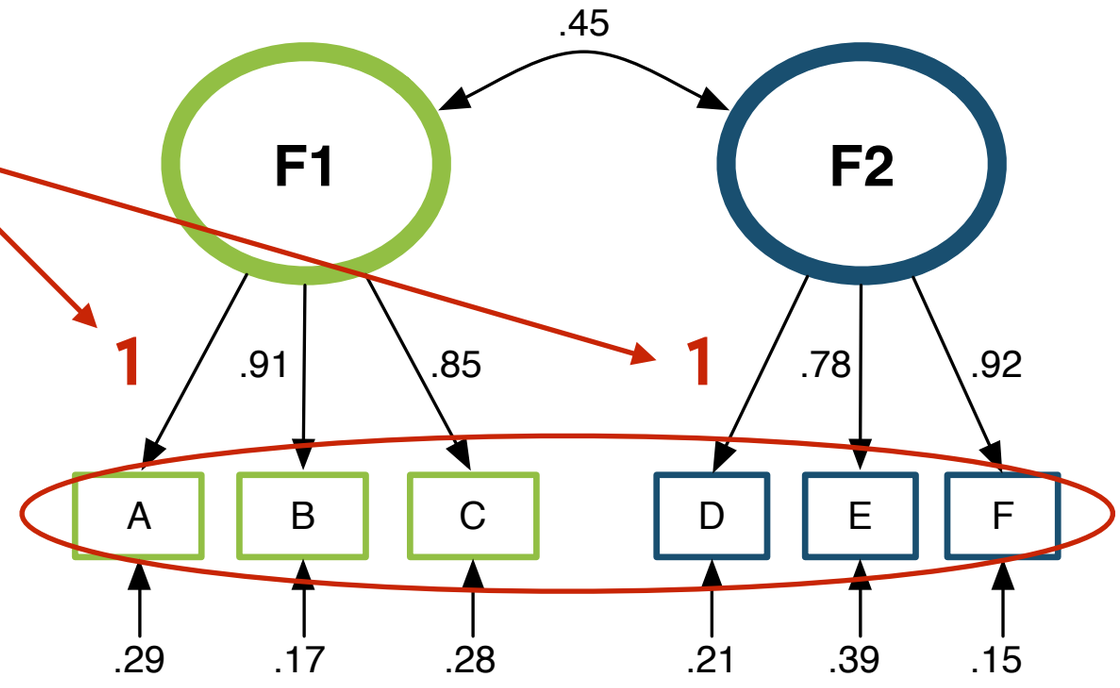


CFA estimation

Unit Loading Identification (ULI)!

```
fit <- cfa(model, data=d)
```

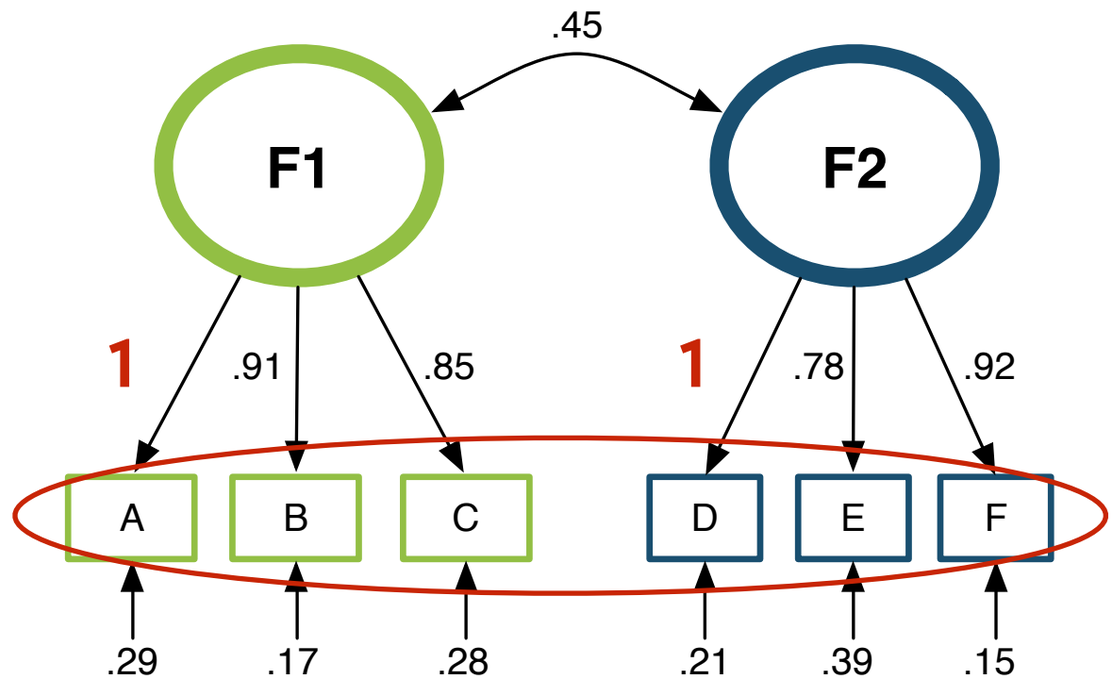
assumed normally distributed ratio variables!



CFA estimation

```
fit <- cfa(model, data=d,
ordered = c("A", "B", "C",
"D", "E", "F"))
```

assumed ordered categorical!

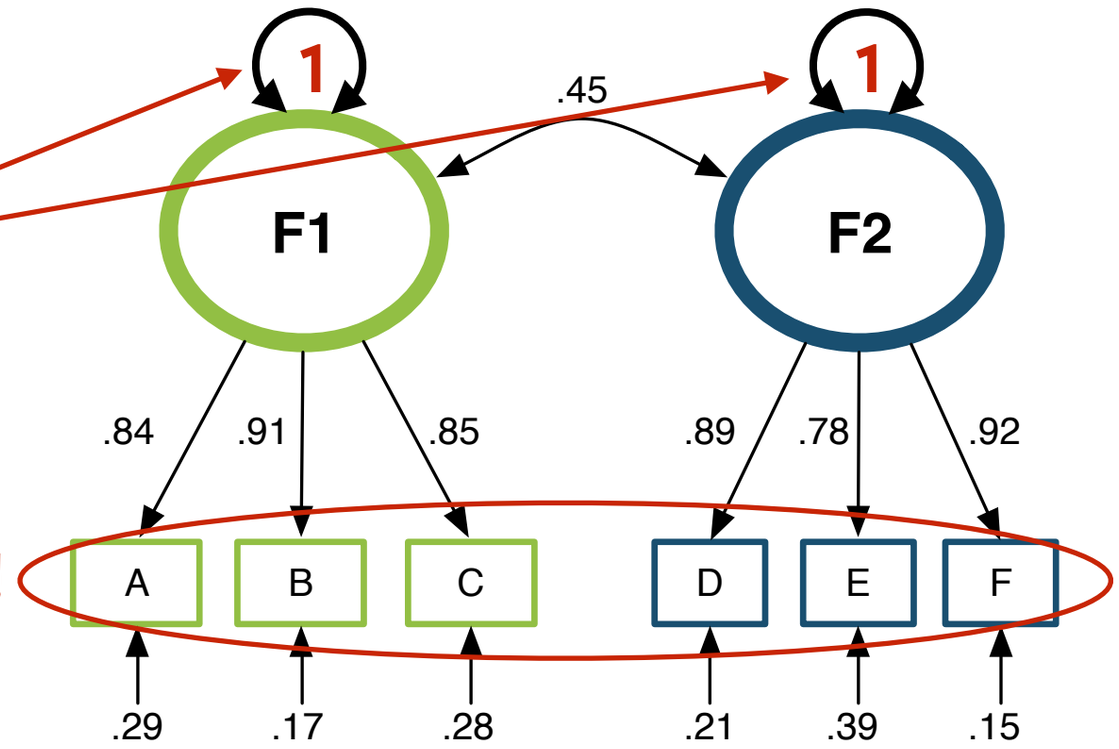


CFA estimation

Unit Variance Identification (UVI)!

```
fit <- cfa(model, data=d,
ordered = c("A", "B", "C",
"D", "E", "F"), std.lv=T)
```

assumed ordered categorical!



CFA output

```
summary(fit, rsquare=T, fit.measures=T)
```

“rsquare” gives us 1-uniqueness values

“fit.measures” gives us CFI, TLI, and RMSEA

Run the CFA

Write model definition:

```
model <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4+c5
underst =~ u1+u2+u3+u4+u5'
```

Run cfa (load package lavaan):

```
fit <- cfa(model, data=twq, ordered=names(twq), std.lv=TRUE)
```

Inspect model output:

```
summary(fit, rsquare=TRUE, fit.measures=TRUE)
```

Run the CFA

Output (model fit):

```
lavaan (0.5-17) converged normally after  39 iterations

  Number of observations                           267

  Estimator                                       DWLS      Robust
  Minimum Function Test Statistic               251.716     365.719
  Degrees of freedom                                224         224
  P-value (Chi-square)                            0.098       0.000
  Scaling correction factor                                   1.012
  Shift parameter                                           117.109
    for simple second-order correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic             48940.029   14801.250
  Degrees of freedom                                253         253
  P-value                                         0.000       0.000
```

Note: we do not really care about this yet
(we should optimize our model first)

Run the CFA

Output (model fit, continued):

```
  User model versus baseline model:

    Comparative Fit Index (CFI)                    0.999       0.990
    Tucker-Lewis Index (TLI)                       0.999       0.989

  Root Mean Square Error of Approximation:

    RMSEA                                          0.022       0.049
    90 Percent Confidence Interval          0.000  0.034       0.040  0.058
    P-value RMSEA <= 0.05                          1.000       0.579

  Weighted Root Mean Square Residual:

    WRMR                                           0.855       0.855

  Parameter estimates:

    Information                                 Expected
    Standard Errors                           Robust.sem
```

Run the CFA

Output (loadings):

	Estimate	Std.err	Z-value	P(>|z|)
Latent variables:				
satisf =~				
s1	0.888	0.018	49.590	0.000
s2	-0.885	0.018	-48.737	0.000
s3	0.771	0.029	26.954	0.000
s4	0.821	0.025	32.363	0.000
s5	0.889	0.018	50.566	0.000
s6	0.788	0.031	25.358	0.000
s7	-0.845	0.022	-38.245	0.000
quality =~				
q1	0.950	0.013	72.421	0.000
q2	0.949	0.013	72.948	0.000
q3	0.942	0.012	77.547	0.000
q4	0.805	0.033	24.257	0.000
q5	-0.699	0.042	-16.684	0.000
q6	-0.774	0.040	-19.373	0.000

These are the loadings (the regression bs on the arrows going from the factor to the item)

They should be > 0.70 (because R^2 = loading^2 should be > 0.5)

Negative loadings are for negative items (please check!!)

Run the CFA

Output (loadings, continued):

```
control =~
    c1                0.712    0.038   18.684    0.000
    c2                0.855    0.024   35.624    0.000
    c3                0.905    0.022   41.698    0.000
    c4                0.723    0.037   19.314    0.000
    c5               -0.424    0.056   -7.571    0.000
underst =~
    u1               -0.557    0.047  -11.785    0.000
    u2                0.899    0.016   57.857    0.000
    u3                0.737    0.030   24.753    0.000
    u4               -0.918    0.016  -58.229    0.000
    u5                0.984    0.010   97.787    0.000
```

Run the CFA

Output (factor correlations):

```
Covariances:
                   Estimate   Std.Err  z-value  P(>|z|)
satisf ~~
  quality             0.686     0.033   20.503    0.000
  control            -0.760     0.028  -26.913    0.000
  underst             0.353     0.048    7.320    0.000
quality ~~
  control            -0.648     0.040  -16.041    0.000
  underst             0.278     0.058    4.752    0.000
control ~~
  underst            -0.382     0.051   -7.486    0.000
```

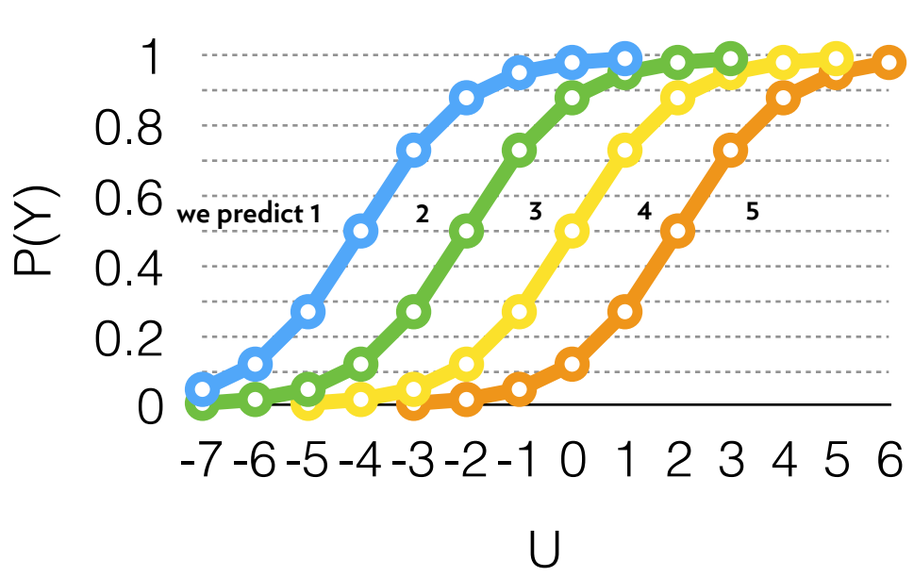
These are the factor correlations (the numbers on the arrows going from one factor to another)
They should not be too high (more about this later)
Note: the control factor turns out to be "lack of control" (that happens sometimes)

Run the CFA

Output (thresholds):

Thresholds:

s1|t1	−1.829	0.148	−12.382	0.000
s1|t2	−1.021	0.093	−10.941	0.000
s1|t3	−0.441	0.080	−5.539	0.000
s1|t4	0.874	0.089	9.874	0.000
s2|t1	−0.330	0.078	−4.207	0.000
s2|t2	0.732	0.085	8.626	0.000
s2|t3	1.157	0.099	11.712	0.000
s2|t4	2.005	0.170	11.790	0.000
s3|t1	−1.737	0.138	−12.581	0.000
s3|t2	−0.834	0.087	−9.540	0.000
s3|t3	−0.222	0.078	−2.869	0.004
s3|t4	1.176	0.100	11.800	0.000
s4|t1	−1.696	0.134	−12.642	0.000
s4|t2	−0.732	0.085	−8.626	0.000
s4|t3	−0.014	0.077	−0.183	0.855
s4|t4	1.037	0.094	11.043	0.000
s5|t1	−1.622	0.128	−12.710	0.000
s5|t2	−0.769	0.086	−8.972	0.000
s5|t3	−0.118	0.077	−1.527	0.127
s5|t4	1.087	0.096	11.339	0.000
s6|t1	−1.737	0.138	−12.581	0.000
s6|t2	−0.902	0.089	−10.094	0.000
s6|t3	0.441	0.080	5.539	0.000
...	...	...	...	...



These are the thresholds for the ordered categorical variables

Run the CFA

Output (variances):

```
Variances:
    s1                  0.212
    s2                  0.218
    s3                  0.406
    s4                  0.326
    s5                  0.210
    s6                  0.379
    s7                  0.286
q1                      0.097
q2                      0.099
q3                      0.112
q4                      0.352
q5                      0.511
q6                      0.401
c1                      0.494
c2                      0.269
c3                      0.180
c4                      0.478
c5                      0.821
u1                      0.690
u2                      0.192
u3                      0.456
u4                      0.157
u5                      0.032
satisf                  1.000
quality                 1.000
control                 1.000
underst                 1.000
```

The variances of the items (observed)

The variances of the factors (fixed to 1, using UVI)

Run the CFA

Output (r-square):

R-Square:

s1	0.788
s2	0.782
s3	0.594
s4	0.674
s5	0.790
s6	0.621
s7	0.714
q1	0.903
q2	0.901
q3	0.888
q4	0.648
q5	0.489
q6	0.599
c1	0.506
c2	0.731
c3	0.820
c4	0.522
c5	**0.179**
u1	**0.310**
u2	0.808
u3	0.544
u4	0.843
u5	0.968

Also called “variance extracted” or “communality”… it is 1 – uniqueness

Should be > 0.50 (or at the very least > 0.40)

Improve the model

Remove items with low communality

check for r-square < 0.40 (or maybe 0.50)

Remove items with high cross-loadings or residual correlations

check the modification indices

Keep at least three items

if necessary, specify a model with cross-loadings or residual correlations... but try to avoid this!

Low communality

Based on r-square, iteratively remove items:

c5 (r-squared = 0.180)

u1 (r-squared = 0.324)

High residuals

High residual correlations:

- The observed correlation between two items is significantly higher (or lower) than predicted
- Might mean that factors should be split up

High cross-loadings:

- When the model suggest that the model fits significantly better if an item also loads on an additional factor
- Could mean that an item actually measures two things

High residuals

In R: modification indices

Modification indices give an **estimate** on how each possible adjustment of the model may improve it

Listed are:

- mi: the modification index (a chi-square value with 1 df)
- epc: the expected value of the parameter if added to the model

High residuals

Get the modification indices

```
mods <- modindices(fit, power=TRUE)
```

Only keep the ones that are significant and large enough to be interesting

```
mods <- mods[grep("\\*", mods$decision),]
```

Display

```
mods
```

High residuals

Look for items involved in several modifications that have a high mi (most important), high epc (less important), or both

Remove the most troublesome one from the model

In this case: u3

Loads on satisfaction and quality, correlates with c1 and s6

Recalculate the modification indices

(etc.)

Improve the model

For all these metrics:

- Remove items that do not meet the criteria, but be careful to keep at least 3 items per factor
- One may remove an item that has values much worse than other items, even if it meets the criteria

(Because of this, I'm going to stop here)

(note: there could be something going on with satisfaction; let's explore later...)

Inspect the model

Inspect the following things in the final model:

Item-fit (this should be good by now)

Factor-fit: Average Variance Extracted

Model-fit: Chi-square test, CFI, TLI, RMSEA

Item-fit

Output (loadings):

```
Latent variables:
  satisf =~
    s1                0.888    0.018   50.049    0.000
    s2               -0.885    0.018  -49.187    0.000
    s3                0.769    0.029   26.847    0.000
    s4                0.822    0.025   32.660    0.000
    s5                0.889    0.017   51.012    0.000
    s6                0.786    0.031   25.139    0.000
    s7               -0.845    0.022  -38.547    0.000
  quality =~
    q1                0.950    0.013   72.301    0.000
    q2                0.950    0.013   73.136    0.000
    q3                0.942    0.012   77.787    0.000
    q4                0.804    0.033   24.346    0.000
    q5               -0.698    0.042  -16.693    0.000
    q6               -0.775    0.040  -19.510    0.000
  control =~
    c1                0.700    0.039   17.958    0.000
    c2                0.859    0.024   36.386    0.000
    c3                0.911    0.022   41.986    0.000
    c4                0.717    0.038   18.773    0.000
  underst =~
    u2                0.910    0.014   63.720    0.000
    u4               -0.922    0.016  -58.796    0.000
    u5                0.984    0.010   93.772    0.000
```

All remaining loadings > 0.70

Item-fit

Output (factor correlations):

```
Covariances:
satisf ~~
  quality           0.687    0.033   20.507    0.000
  control          -0.762    0.029  -26.711    0.000
  underst           0.315    0.052    6.105    0.000
quality ~~
  control          -0.646    0.041  -15.718    0.000
  underst           0.263    0.059    4.494    0.000
control ~~
  underst          -0.328    0.058   -5.681    0.000
```

Item-fit

Output (r-square):

```
R-Square:

     s1                  0.788
     s2                  0.783
     s3                  0.592
     s4                  0.675
     s5                  0.791
     s6                  0.617
     s7                  0.714
     q1                  0.902
     q2                  0.902
     q3                  0.887
     q4                  0.646
     q5                  0.487
     q6                  0.601
     c1                  0.490
     c2                  0.738
     c3                  0.830
     c4                  0.514
     u2                  0.828
     u4                  0.849
     u5                  0.968
```

A few are < 0.50, but all are > 0.48, so this is quite okay

Factor-fit

Average Variance Extracted (AVE)

In lavaan output: average of R-squared per factor

Convergent validity:

AVE > 0.5

Discriminant validity

$\sqrt{}$(AVE) > largest correlation with other factors

Factor-fit

Satisfaction:

AVE = 0.709, $\sqrt{(AVE)}$ = 0.842, largest correlation = 0.762

Quality:

AVE = 0.737, $\sqrt{(AVE)}$ = 0.859, largest correlation = 0.687

Control:

AVE = 0.643, $\sqrt{(AVE)}$ = 0.802, largest correlation = 0.762

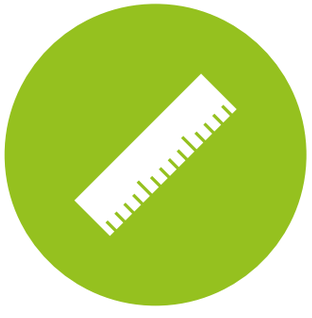
Understandability:

AVE = 0.874, $\sqrt{(AVE)}$ = 0.935, largest correlation = 0.341

Model-fit metrics

Chi-square test of model fit:

- Tests whether there any significant misfit between estimated and observed correlation matrix
- Often this is true ($p < .05$)... models are rarely perfect!
- Alternative metric: chi-squared / df < 3 (good fit) or < 2 (great fit)



Model-fit metrics

CFI and TLI:

- Relative improvement over baseline model; ranging from 0.00 to 1.00
- CFI should be > 0.96 and TLI should be > 0.95

RMSEA:

- Root mean square error of approximation
- Overall measure of misfit
- Should be < 0.05, and its confidence interval should not exceed 0.10.

Model-fit metrics

Output (model fit):

```
lavaan (0.5-17) converged normally after  38 it

  Number of observations

                                                  DWLS      Robust
  Estimator
  Minimum Function Test Statistic               162.211     286.057
  Degrees of freedom                                164         164
  P-value (Chi-square)                            0.525       0.000
  Scaling correction factor                                   0.755
  Shift parameter                                            71.330
    for simple second-order correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic             46290.833   14383.462
  Degrees of freedom                                190         190
  P-value                                         0.000       0.000
```

Model shows significant misfit, but Chi-square / df is good: 286 / 164 = 1.76

This tests if the model is better than the worst possible model (unsurprisingly, it is…)

Run the CFA

Output (model fit, continued):

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000       0.991
  Tucker-Lewis Index (TLI)                       1.000       0.990

Root Mean Square Error of Approximation:

  RMSEA                                          0.000       0.053
  90 Percent Confidence Interval          0.000  0.027       0.043  0.063
  P-value RMSEA <= 0.05                          1.000       0.311

Weighted Root Mean Square Residual:

  WRMR                                           0.777       0.777

Parameter estimates:

  Information                                 Expected
  Standard Errors                           Robust.sem
```

CFI and TLI are excellent

RMSEA = .053 is not great, but the 90% CI is ok: [.043, .063] (not > .10)

You can ignore WRMR

Summary

Specify and run your CFA

Alter the model until all remaining items fit

Make sure you have at least 3 items per factor!

Report final loadings, factor fit, and model fit

Summary

We conducted a CFA and examined the validity and reliability scores of the constructs measured in our study.

Upon inspection of the CFA model, we removed items c5 (communality: 0.180) and u1 (communality: 0.324), as well as item u3 (high cross-loadings with several other factors). The remaining items shared at least 48% of their variance with their designated construct.

Summary

To ensure the convergent validity of constructs, we examined the average variance extracted (AVE) of each construct. The AVEs were all higher than the recommended value of 0.50, indicating adequate convergent validity.

To ensure discriminant validity, we ascertained that the square root of the AVE for each construct was higher than the correlations of the construct with other constructs.

Summary

Construct	Item	Loading
System satisfaction Alpha: 0.92 AVE: 0.709	I would recommend TasteWeights to others. TasteWeights is useless. TasteWeights makes me more aware of my choice options. I can make better music choices with TasteWeights. I can find better music using TasteWeights. Using TasteWeights is a pleasant experience. TasteWeights has no real benefit for me.	0.888 -0.885 0.768 0.822 0.889 0.786 -0.845
Perceived Recommendation Quality Alpha: 0.90 AVE: 0.737	I liked the artists/bands recommended by the TasteWeights system. The recommended artists/bands fitted my preference. The recommended artists/bands were well chosen. The recommended artists/bands were relevant. TasteWeights recommended too many bad artists/bands. I didn't like any of the recommended artists/bands.	0.950 0.950 0.942 0.804 -0.697 -0.775
Perceived Control Alpha: 0.84 AVE: 0.643	I had limited control over the way TasteWeights made recommendations. TasteWeights restricted me in my choice of music. Compared to how I normally get recommendations, TasteWeights was very limited. I would like to have more control over the recommendations. I decided which information was used for recommendations.	0.700 0.859 0.911 0.716
Understandability Alpha: 0.92 AVE: 0.874	The recommendation process is not transparent. I understand how TasteWeights came up with the recommendations. TasteWeights explained the reasoning behind the recommendations. I am unsure how the recommendations were generated. The recommendation process is clear to me.	0.893 -0.923 0.987

Summary

	Alpha	AVE	Satisfaction	Quality	Control	Underst.
Satisfaction	0.92	0.709	**0.842**	0.687	–0.762	0.336
Quality	0.90	0.737	0.687	**0.859**	–0.646	0.282
Control	0.84	0.643	–0.762	–0.646	**0.802**	–0.341
Underst.	0.92	0.874	0.336	0.282	–0.341	**0.935**

diagonal: √(AVE)

off-diagonal: correlations

Alternative models

s3 and s4 are more highly correlated, so:

```
emodel <- 'satisf =~ s1+s2+s5+s6+s7
choice =~ s3+s4
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4+c5
underst =~ u1+u2+u3+u4+u5'
```

Run cfa:

```
efit <- cfa(emodel, data=twq, ordered=names(twq), std.lv=T)
```

Inspect model output:

```
summary(efit, rsquare=TRUE, fit.measures=TRUE)
```

Factor-fit

Satisfaction: AVE = 0.744, $\sqrt{(AVE)}$ = 0.863

Choice satisfaction: AVE = 0.782, $\sqrt{(AVE)}$ = 0.884

Correlation between them = 0.889

Conclusion: no discriminant validity!

Alternative models

s3 and s4 are more highly correlated, so:

```
fmodel <- 'satisf =~ s1+s2+s3+s4+s5+s6+s7
quality =~ q1+q2+q3+q4+q5+q6
control =~ c1+c2+c3+c4+c5
underst =~ u1+u2+u3+u4+u5
s3 ~~ s4'
```

Run cfa and inspect output:

```
ffit <- cfa(emodel, data=twq, ordered=names(twq), std.lv=T)
summary(ffit, rsquare=TRUE, fit.measures=TRUE)
```

“It is the mark of a truly intelligent person
to be moved by statistics.”

T H A N K S !

George Bernard Shaw